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The major accomplishments of this grant are:

1) Worked out a model and a computer program for the motion of polycrystalline interface in 2-d.

2) Organized the AMS Special Session on computing Optimal Geometries, at the annual meeting in san Francisco (jointly with Fred Almgren and Al Marden), and edited the proceedings.

3) Worked out some of the theroy for motion of curves in 3-d by crystalline curvature, and implemented some if it in a computer program.

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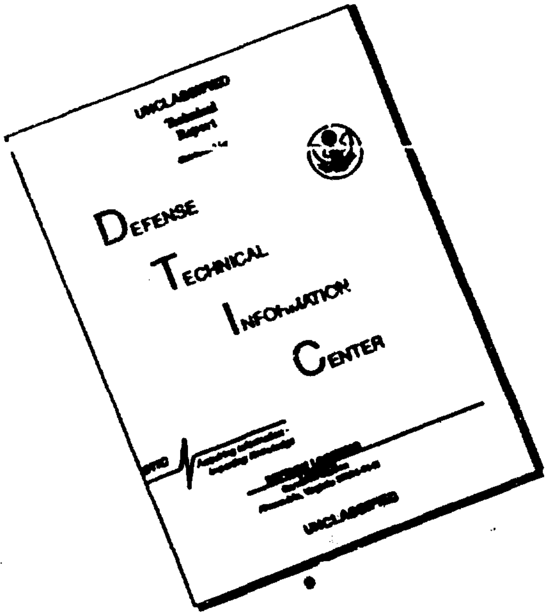

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FINAL REPORT
GRANT AFOSR-87-0277
JEAN E. TAYLOR

Since March 1990, my major efforts have included:

1) Working out a model and a computer program for the motion of polycrystalline interfaces in 2-d, in the crystalline case where the normal velocity of a segment S_i is of the form

$$-M(\mathbf{n}(S_i))(\Omega + wmc(S_i)).$$

Here $\mathbf{n}(S_i)$ is the oriented unit normal of S_i, Ω is a constant, and *wmc* stands for "weighted mean curvature." *wmc* is defined to be the rate of increase in surface energy with volume under deformations consisting of changing the distance of S_i from the origin while keeping its same orientation and adjacencies to other segments. A major question, which I have solved during the past year, is how to handle fixed boundaries and triple junctions.

I have written one substantial paper, with complete definitions and proofs, plus two short summary papers on this subject this year:

(i) *Motion of Curves by Crystalline Curvature, Including Triple Junctions and Boundary Points*, to appear in the Proceedings of the AMS Summer Institute on Differential Geometry, held in 1990. This is the definitive paper.

(ii) *Motion by Crystalline Curvature*, to appear in Computing Optimal Geometries, Proceedings of an AMS Special Session.

(ii) *Crystalline Geometric Crystal Growth*, to appear in Proceedings of Workshop on Theoretical and Numerical Aspects of Geometric Variational Problems, Australian National University, Sept. 1990.

I also made a seven-and-a-half minute videotape on *Motion by Crystalline Curvature*, which will appear as part of Computing Optimal Geometries, the proceedings of the AMS Special Session mentioned above.

2) Organizing the AMS Special Session on Computing Optimal Geometries, at the annual meeting in San Francisco (jointly with Fred Almgren and Al Marden), and editing its proceedings. Since this proceedings includes videotapes for the first time (for the AMS and for me), this editing has involved paying attention to a host of details. I also made major efforts to make things happen quickly, since the videotapes are expected to be of the greatest value if they appear quickly.

3) Writing the paper *Geometric Models of Crystal Growth* with John Cahn and Carol Handwerker. This paper surveys nine different mathematical methods for trying to solve the problem of geometric crystal growth, outlining the methods and giving the circumstances in which each performs best. It also surveys all computational methods of which

we are aware. This paper has involved considerable effort on my part to understand the various methods, many of which I was not previously familiar with, and to interpret them for my colleagues. This paper is not quite completed, though we hope to finish it soon; the attempt to make it accessible to materials scientists as well as mathematically accurate has involved multiple revisions. Mathematicians as well as materials scientists are eagerly requesting copies, and I will use it as the basis for an advanced graduate course in fall 1991.

4) Writing the paper *Destabilization of the Tetrahedral Point Junction by Positive Triple Junction Line Energy*, with Frank Morgan. This paper will appear in Scripta Metallurgica et Materialia; it points out that the standard tetrahedral point junction, where four regions meet at a point, is no longer energy-minimizing when a junction line energy is included in the total energy along with surface area. Rather, a surface with a line segment along which four surfaces meet is shown to have less total energy. An estimate is made for the length that such a four-fold junction line might be expected have, based on estimates for line junction energy that are given by dislocation energies; this distance is of the order of several atomic dimensions and might therefore be seen in experiments.

5) Writing the paper *The Motion of Multiple-Phase Junctions under Prescribed Phase-Boundary Velocities*. This paper has not yet been completed, although a substantially complete draft exists. It shows that the motion of triple junctions can in general uniquely defined, using Huygen's principle of least time, for three arbitrary normal velocity functions and three arbitrary half lines meeting at a point. This motion can be found by the appropriate use of characteristics, with something like "refraction" of the characteristics when the growth is first through one phase and then another. There are important exceptions where there is no solution, and where for certain initial configurations, there is non-uniqueness (although once growth has started using any of the possibilities, it is then unique for all subsequent time).

6) Working out some of the theory for motion of curves in 3-d by crystalline curvature, and implementing some of it in a computer program. An example of the use of the program is at the end of the videotape mentioned in (1) above; in particular, at this time it handles surfaces with fixed polygonal boundaries, where the initial surface has no two adjacent convex or concave corners. Extension of the theory and program to the general case is one of my highest priorities.